Heat Transfer

For

Mechanical Engineering

By

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Syllabus for Heat Transfer

Modes of heat transfer, one dimensional heat conduction, resistance concept, electrical analogy, unsteady heat conduction, fins; dimensionless parameters in free and forced convective heat transfer, various correlations for heat transfer in flow over flat plates and through pipes; thermal boundary layer; effect of turbulence; radiative heat transfer, black and grey surfaces, shape factors, network analysis; heat exchanger performance, LMTD and NTU methods.

Previous Year GATE Papers and Analysis

GATE Papers with answer key

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“It's fine to celebrate success but it is more important to heed the lessons of failure”

.... Bill Gates

CHAPTER 1

Conduction

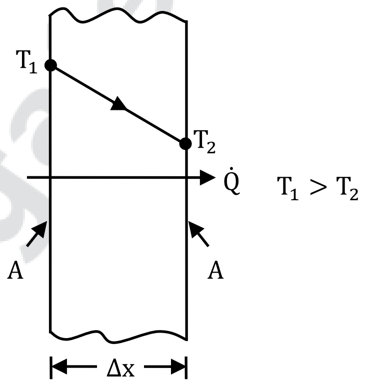
Learning Objectives

After reading this chapter, you will know:

1. Modes of Heat Transfer, Thermal Conductivity, Thermal Diffusivity
2. One Dimensional Heat Conduction, Conduction Through Cylindrical Wall
3. Conduction Through Sphere, Thermal Resistance
4. Lumped System Method, Critical Radius of Insulation
5. Heat Transfer Through Fins, Fin Efficiency, Fin Effectiveness

Introduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids or gases. Conduction is due to the collisions and diffusion of the molecules during their random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.



Heat Conduction Through a Large Plane Wall of Thickness, Δx and Area, A

Consider steady heat conduction through a large plane wall of thickness $\Delta x = L$ and area A, as shown in figure. The temperature difference across the wall is $\Delta T = T_2 - T_1$

$$\text{Rate of heat conduction} \propto \frac{\text{(Area) (Temperature)}}{\text{Thickness}}$$

Mathematically,

$$\dot{Q}_{cond} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x}$$

Where the constant of proportionality k is the thermal conductivity of the material, (differential form)

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$

Which is called Fourier's law of heat conduction, here dt/dx is the temperature gradient, which is the slope of the temperature curve on a T-x diagram (the rate of change of T with x) at location x. The relation above indicates that the rate of heat conduction in a direction is proportional to the temperature gradient in that direction. Heat is conducted in the direction of decreasing temperature.

Thermal Conductivity

The thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat.
In SI unit of thermal conductivity is W/mK

Thermal Diffusivity

Material property that appears in the transient heat conduction analysis is the thermal diffusivity. Which represents how fast heat diffuses through a material and is defined as,

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p}$$

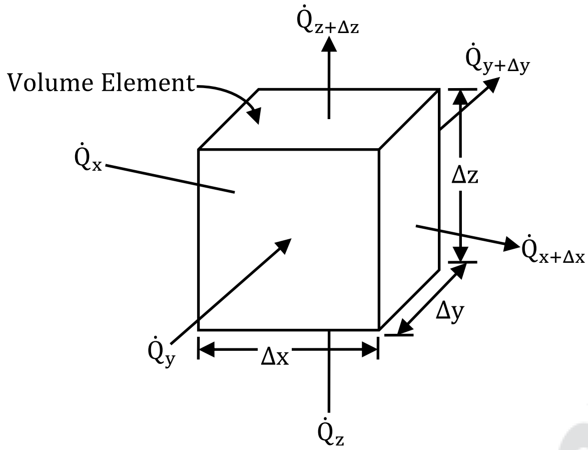
Note: That the thermal conductivity, k represents how well a material conducts heat and the heat capacity, ρC_prepresents how much energy a material stores per unit volume. Therefore, the thermal diffusivity of a material can be viewed as the ratio of the heat conducted through the material to the heat stored per unit volume. A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium.
SI unit of thermal diffusivity is m^2/s

One Dimensional Heat Conduction

Heat transfer has direction as well as magnitude. The rate of heat conduction in a specified direction is proportional to the temperature gradient, which is the change in temperature per unit length in that direction. Heat conduction in a medium, in general, is three-dimensional and time dependent. That is, $T = T(x, y, z, t)$ and the temperature in a medium varies with position as well as time. Heat conduction in a medium is said to be steady when the temperature does not vary with time and unsteady or transient when it does. Heat conduction in a medium is said to be one-dimensional when conduction is significant in one direction only and negligible in the other two dimensions, the governing differential equation in such systems in rectangular, cylindrical and spherical coordinate systems is derived in below section.

Rectangular Co-ordinates

Consider a small rectangular element of length Δx, width Δy and height Δz, as shown in figure. Assume the density of the body is ρ and the specific heat is C, an energy balance on this element during a small time interval Δt can be expressed as



Three-Dimensional Heat Conduction Through a Rectangular Volume Element

$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction at}\\ \text{x, y and z}\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x, y+\Delta y\\ z+\Delta z\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{array}\right)$$

Noting that the volume of the element is $V_{Element} = \Delta x \Delta y \Delta z$, the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C \times \frac{\partial T}{\partial t}$$

Since, from the definition of the derivative and Fourier's law of heat conduction.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{Fourier – Biot equation}$$

1. Steady-State: (Poisson's equation)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0$$

2. Transient, no heat generation: (Diffusion equation)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

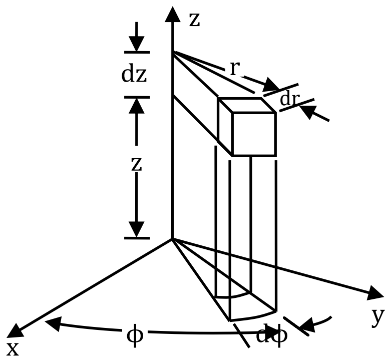
3. Steady state, no heat generation: (Laplace equation)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\nabla^2 T = 0$$

Cylindrical Co-ordinates

$x = r\cos\phi,\ \ y = r\sin\phi$ and $z = z$



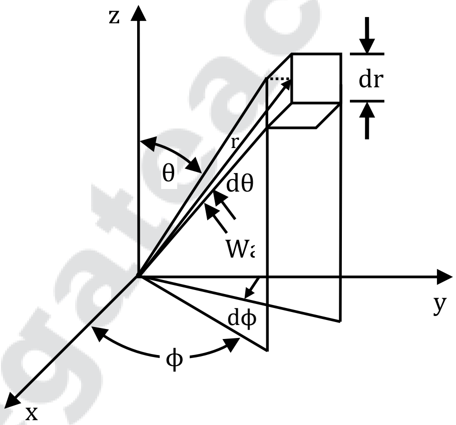
A differential Volume Element in Cylindrical Co-ordinates

After lengthy manipulations we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t}$$

Spherical Co-ordinates

$x = r\cos\phi\sin\phi,\ y = r\sin\phi\sin\theta$ and $z = \cos\theta$



A Differential Volume Element in Spherical Co-ordinates

Again after lengthy manipulations, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{r^2\,\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t}$$

Conduction Through a Cylindrical Wall

For a cylinder at steady state, with no internal heat generation, the equation becomes

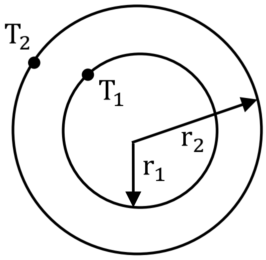
$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=0 \Rightarrow \left(\frac{1}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0 \text{ and } K=\text{contant}$$

B.C → Boundary conditional

On integration and substituting the B.C's, we get the temperature distribution equation as

$$\frac{T - T_1}{T_2 - T_1} = \frac{\log(r/r_1)}{\log(r_2/r_1)}$$

$Q = (T_1 - T_2)/R_1$ where, $R_1 = \dfrac{\log(r_2/r_1)}{2\pi kL}$



Conduction Through a Cylindrical Wall

Comparing the above equation to that of heat transfer through a wall

$Q = (kA\,(T_1 - T_2))/\delta$

$\quad = kA_m\,(T_1 - T_2)/(r_2 - r_1)$

Where, A_m is the logarithmic mean area $= (A_2 - A_1)/\log(A_2 / A_1)$

Conduction Through Sphere

Steady state, one dimensional with no heat generation equation in spherical co-ordinates is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dt}{dr}\right) = 0$$

On integration and substituting the B.C's we get the temperature distribution equation as,

$$\frac{T - T_1}{T_1 - T_2} = \frac{r_2}{r}\left[\frac{r - r_1}{r_2 - r_1}\right]$$

From the above, it is seen that it is hyperbolic

Heat Transfer rate $= Q = \dfrac{(T_1 - T_2)4\pi k r_1 r_2}{(r_2 - r_1)} = \left[\dfrac{T_1 - T_2}{R_t}\right]$

Where, $R_t = \{(r_2 - r_1) / 4\pi\,k\,r_1 r_2\}$

Comparing the above equation to that of heat transfer through a wall.

$Q = (kA\,\Delta T / \delta) = \{(k\,A_m\,\Delta T) / (r_2 - r_1)\}$

Where, $A_m = 4\pi r_1 r_2 = 4\pi r_m^2$

$\qquad r_m = \sqrt{r_1 r_2}$

Conduction Through	Variation of Temperature
Plane wall	Linear
Cylinder	Logarithmic
Sphere	Hyperbolic

Thermal Resistance

In particular, there exists an analogy between the diffusion of heat and electrical charge. Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance may be associated with the conduction of heat.

$$R_{t,cond} = \frac{T_{s1} - T_{s2}}{q_x} = \frac{L}{kA}$$

Similarly, for electrical conduction in the same system, Ohm's law provides an electrical resistance of the form

$$R_e = \frac{E_{s1} - E_{s2}}{1} = \frac{L}{\sigma A}$$

The analogy between above equations is obvious. A thermal resistance may also be associated with heat transfer by convection at a surface. From Newton's law of cooling.

$$q = hA\,(T_s - T_\infty)$$

The thermal resistance for convection is then

$$R_{total,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

Circuit representations provide a useful tool for both conceptualizing and quantifying heat transfer problems. The equivalent thermal circuit for the plane wall with convection surface conditions is shown in figure.

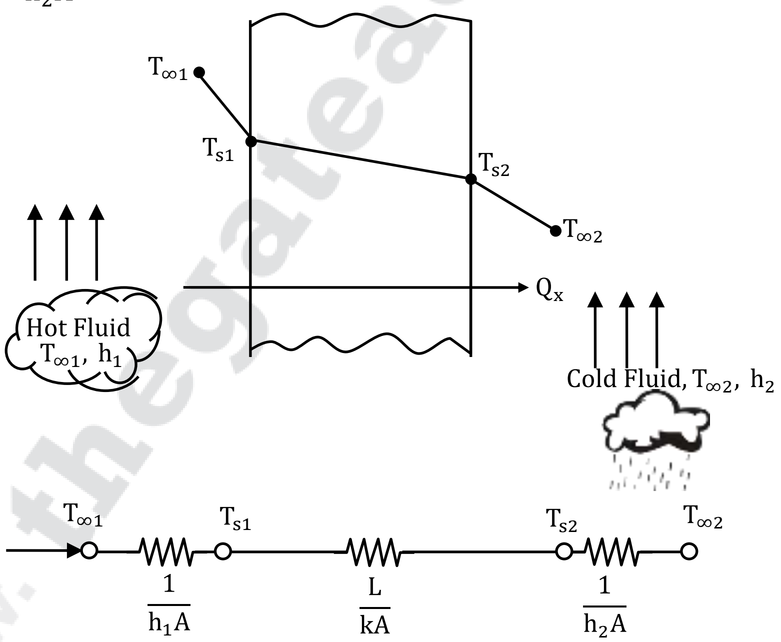
$$Q_x = \frac{T_{\infty 1} - T_{s1}}{1/h_1 A} = \frac{T_{s1} - T_{s2}}{L/h_1 A} = \frac{T_{s2} - T_{\infty 2}}{1/h_2 A}$$

In terms of the overall temperature difference, $T_{\infty_1} - T_{\infty_2}$ and the total thermal resistance, R_{tot}, the heat transfer rate may also be expressed as,

$$Q_x = \frac{T_{\infty_1} - T_{\infty_2}}{R_{Total}}$$

Because the conduction and convection resistances are in series and may be summed, it follows that

$$R_{total} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$



Equivalent Thermal Circuits

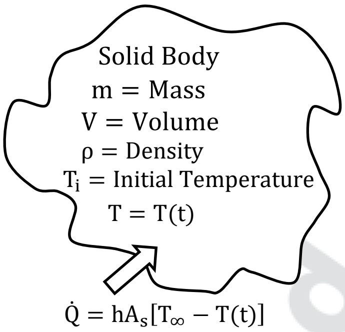
Radiation exchange between the surface and surroundings may also be important if the convection heat transfer coefficient is small (as it often is for a natural convection in a gas). A thermal resistance for radiation may be defined by

$$R_{total,rad} = \frac{T_s - T_{sur}}{q_{rad}}$$

Lumped System Method

The first method of analysis is using lumped system method. Lumped systems are systems in which the temperature of a solid varies with time but remains uniform throughout the solid at any time.

In heat transfer analysis, some bodies are observed to behave like a "lump" whose interior temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, T(t). Heat transfer analysis that utilizes this idealization is known as lumped system analysis, which provides great simplification in certain classes of heat transfer problems without much sacrifice from accuracy,



The Geometry and Parameters Involved in the Lumped System Analysis

Consider a body of arbitrary shape of mass, m, volume, V, surface area, A_s, density, ρ and specific heat, C_p, initially at a uniform temperature, T_i at time, t = 0, the body is placed into a medium at temperature T_∞ and heat transfer takes place between the body and its environment, with a heat transfer coefficient, h. For the sake of discussion, we will assume that $T_\infty > T_i$, T_i but the analysis is equally valid for the opposite case. We assume lumped system analysis to be applicable, so that the temperature remains uniform within the body at all times and changes with time only, T = T(t)

$$\begin{pmatrix}\text{Heat transfer into the body} \\ \text{during dt}\end{pmatrix} = \begin{pmatrix}\text{The increase in the} \\ \text{energy of the body} \\ \text{during dt}\end{pmatrix}$$

Or

$$hA_s(T_\infty - T)dt = mC_p dT$$

Noting that $m = \rho V$ and $dT = d(T - T_\infty)$ since T_∞ = constant,

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V C_p} dt$$

Integrating from t = 0, at which$T = T_i$, to any time, t, at which$T = T(t)$, gives

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V C_p} t$$

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t) - T_\infty}{T_1 - T_\infty} = e^{-bt}$$

Where,

$$b = \frac{h\,A_s}{\rho V C_p}$$